The Predictive Performance of Asymmetric Normal Mixture GARCH in Risk Management: Evidence from Turkey

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Abstract

The purpose of this study is to test predictive performance of Asymmetric Normal Mixture GARCH (NMAGARCH) and other GARCH models based on Kupiec and Christoffersen tests for Turkish equity market. The empirical results show that the NMAGARCH perform better based on %99 CI out-of-sample forecasting Christoffersen test where GARCH with normal and student-t distribution perform better based on %95 Cl out-of-sample forecasting Christoffersen test and Kupiec test. These results show that none of the model including NMAGARCH outperforms other models in all cases as trading position or confidence intervals and the real implications of these results for Value-at-Risk estimation is that volatility model should be chosen according to confidence interval and trading positions. Besides, NMAGARCH increases predictive performance for higher confidence internal as Basel requires.

Key Words: *GARCH, Asymmetric Normal Mixture GARCH, Christoffersen Test, Emerging Markets*
JEL Codes: *C52, C32, G0*

Özet - Risk Yönetiminde Asimetrik Normal Karma GARCH Modelinin Öngörü Performansı: Türkiye Uygulaması

Bu çalışmanın amacı, Türk hisse senedi piyasası için Asimetrik Normal Karma GARCH (NMAGARCH) ve diğer GARCH modellerinin öngörü performansını Kupiec ve Christoffersen geriye dönük testleri ile test etmektir. Ampirik bulgular %99 güven aralığı için örneklem dışı Christoffersen testine göre NMAGARCH modelinin, %95 güven aralığı için örneklem dışı Christoffersen ve Kupiec testlerine göre normal ve student-t dağılımlı GARCH modelinin diğer modellerden daha iyi sonuç verdiğini göstermektedir. Bu sonuçlar, NMAGARCH modeli de dâhil olmak üzere hiçbir modelin diğer modellere göre tüm posizyon ve güven aralıklarında daha iyi sonuç vermediğini göstermektedir ve Riske Maruz Değer hesaplamasında bu bulgunun sonucu volatilite modelinin ticaret posizyonu ve güven aralığına göre seçilmesi gerektiğidir. Ayrıca, NMAGARCH modeli Basel'ında gerektirdiği şekilde yüksek güven aralığında öngörü performansını arttırmaktadır.

Anahtar Kelimeler: *GARCH, Asimetrik Normal Karma GARCH, Christoffersen Testi, Gelişmekte Olan Piyasalar*
JEL Sınıflaması: *C52, C32, G0*

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1. Introduction

Modeling return volatility of the financial instruments is a crucial task for risk management, trading and hedging strategies. Especially in the developing markets in which non-linear behaviors in stock returns and asymmetries in the return volatilities occur due to dynamic and chaotic financial environment, advanced financial modeling techniques are required for accurate and correct estimation of return volatility.

In emerging markets, because of portfolio investments of hedge funds, low market volume and unstable political and economic conditions, the volatility in the returns of financial variables are relatively higher and shows an asymmetric character in that it increases in case of emergence of negative information. What is more, high volatility in the form of shocks causes regime switches, which are not easy to be estimated and modeled with static econometric models.

In the finance literature, among many volatility models, the most successful models are seen as the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models by Bollerslev (1986), who generalizes the seminal idea on Autoregressive Conditional Heteroskedasticity (ARCH) by Engle (1982), and their numerous generalizations that add asymmetries, long memory, or structural breaks. GARCH models are popular due to their ability to capture many of the typical stylized facts of financial time series, such as time-varying volatility, persistence and volatility clustering. Andersen and Bollerslev (1998) find that GARCH models do really provide good volatility forecasts, in particular when a good proxy for the latent volatility, such as the realized volatility, is adopted.

In this paper, five main GARCH models are used to estimate the stock market volatility. In addition, each model is applied on the time series with different normality assumptions, mainly normal distribution, Student's t distribution and skewed Student's t distribution. In recent research, asymmetric normal mixture GARCH models have been used in volatility modeling. Research by Alexander and Lazar (2003, 2005, 2006) uses GARCH(1,1) models with normal mixture conditional densities having flexible individual variance processes and time-varying conditional higher moments.

The importance of using (asymmetric) normal mixture GARCH process lies in the fact that it can captures tails in the financial time series more properly. That is very important for modeling return volatility in the emerging financial markets where asymmetric high volatility observed during financial shocks. The emerging markets are open to internal or external shocks observed due to hot money movements, low trade volume, thin trading and instability. Markov regime switching models are used to capture the effects of the sudden shocks in the emerging markets. The normal mixture GARCH models are similar to Markov switching models and easier for use as it will be explained in the methodology part. This paper tries to estimate the return volatility in the Istanbul Stock Exchange by using five GARCH models including the normal mixture GARCH models with tree different normality distributions. The aim of this research is to examine if the normal mixture GARCH models produce more accurate results and are able to capture shocks as long memory processes.

The paper is constructed as follows. In the next part, a literature review on the predictive performance in return volatility in financial markets is presented. The test results in the literature with different markets and sample periods are compared. In the third part, the methodologies of the GARCH models and different normality distributions are introduced. The importance is given on the methodology of asymmetric normal mixture GARCH model introduced by Alexander and Lazar (2003, 2005). After presenting the descriptive statistics of the data, empirical tests and Kupiec and Christoffersen back-tests are implemented. The predictive performances of the fifteen GARCH models in-sample and out-of-sample forecasting results are compared. The paper ends with suggestions for risk management and trading functions for their Value-at-Risk calculations and future financial research conducted in the transitory economics.

2. Literature Review

Early empirical evidence has shown that a high ARCH order should be used to capture the dynamics of the conditional variance. GARCH process constructed by Bollerslev (1986) solves the problem in the ARCH model. GARCH model is based on an infinite ARCH specification and reduces the number of estimated parameters by imposing non-linear restrictions on them.

The GARCH models are extended under different motivation and assumption by researchers. The alternative models are Exponential GARCH (EGARCH, Nelson(1991)), GJR (Glosten, Jagannathan, and Runkle, 1993), Asymmetric Power ARCH (APARCH, Ding, Granger, and Engle (1993)), Integrated GARCH (IGARCH, Engle and Bollerslev (1986)), Fractionally Integrated GARCH (FIGARCH, Baillie, Bollerslev, and Mikkelsen (1996) and Chung (1999)), Fractionally Integrated Exponential GARCH (FIEGARCH, Bollerslev and Mikkelsen (1996)), Fractionally Integrated Asymmetric Power ARCH (FIAPARCH, Tse (1998)) and Hyperbolic GARCH

(HYGRACH, Davidson (2001)). Ackert and Racine (1999), Darrat and Benkato (2003) and Puttonen (1995) use different GARCH models with different markets and time periods and conclude that the GARCH models are successful to model the volatility in the stock returns. As a long memory process, the normal mixture GARCH model captures shocks effects in the time series is used by Alexander and Lazar (2005, 2006).

The GARCH models are used with different assumptions on normality distributions. Bollerslev and Wooldridge (1992) shows that under the normality assumption, the quasi maximum likelihood estimator is consistent if the conditional mean and the conditional variance are correctly specified. This estimator is, however, inefficient with the degree of inefficiency increasing with the degree of departure from normality. Since the issue of fat-tails is crucial in empirical finance, using a more appropriate distribution might reduce the excess kurtosis displayed by the residuals of conditional heteroscedasticity models. Palm (1996), Pagan (1996) and Bollerslev, Chou, and Kroner (1992) use fat-tailed distributions in the literature. Bollerslev (1987), Hsieh (1989), Baillie and Bollerslev (1989) and Palm and Vlaar (1997) show that these distributions perform better in capturing the higher observed kurtosis.

The importance of skewness is explained in many researches. In a recent study, Christoffersen and Jacobs (2004) show that a simple asymmetric GARCH, that captures the leverage effect, performs best of all GARCH model considered. Bekaert and Wu (2000) and Wu (2001) display the fact that the 'leverage effect' in stocks determines a strong negative correlation between returns and volatility, which is the most important reason for skewness in stock returns. Christoffersen, Heston and Jacobs (2004), Bates (1991) focus on the connection between time-variability in the physical conditional skewness and the empirical characteristics of option implied volatility skews.

The difference between the physical and risk neutral skews is among the recent issues in financial research. Bates (2003) states that the difference between the risk-neutral and observed distributions cannot be explained unless the existence of a time-varying volatility risk premium is considered. Bates (2003) conducts the research based on real-world models with a single volatility component. However, Haas, Mittnik and Paolella (2004) and Alexander and Lazar (2005) show that GARCH models with time-varying volatility provide a better fit to the physical conditional densities than GARCH specifications with only one volatility state. The conditional higher moments endogenously determined are time varying in those models. Therefore, their implied volatility skews exhibit the features of risk neutral index skews.

Non-normality in conditional and unconditional returns is higher than that can be captured by GARCH(1,1) models with normally distributed errors. Bollerslev (1987) constructs GARCH(1,1) model with Student-t distribution. Fernandez and Steel (1998) extend the model to the skewed t-distribution. These t-GARCH models have no time-variation in the conditional higher moments. On the other hand, Haas, Mittnik and Paolella (2004) and Alexander and Lazar (2006) in their recent researches conduct GARCH(1,1) models with normal mixture conditional densities. The normal mixture GARCH models are flexible in individual variance processes and have time-varying conditional higher moments. Alexander and Lazar (2006) show that if the model has more than two variance components, biases in parameter estimates are likely to result, and the estimated conditional skewness and excess kurtosis can be unstable over time. For modeling major exchange rate time-series, they find that the mixture of two GARCH(1,1) components models outperform both symmetric and asymmetric t-GARCH models and normal mixture GARCH(1,1) models with more than two components.

For stock market returns volatility, there are certain discrete time-varying models in the literature based on asymmetric GARCH models Engle and Ng (1993), Glosten, Jagahannathan, and Runkle (1993) Nelson (1991) show that the models capture only one source of skewness, namely, the leverage effect. Additional structure is needed to capture the empirical observations about the nature of skewness in the risk-neutral equity index skew. This paper deals with the problem by using asymmetric normal mixture GARCH model with reality check.

3. Methodology

Reliable forecasting of return volatility in the financial markets is crucial for trading, risk management and derivative pricing. Return volatility is affected by time dependent information flows resulting in pronounced temporal volatility clustering. Therefore, financial time series should be parameterized with Autoregressive Conditional Heteroskedastic (ARCH) models modeling a time-turn varying conditional variance as a linear function of past squared residuals and of its past values. In other words, ARCH models are used to forecast conditional variances in that the variance of the dependent variable is modeled as a function of past values of the dependent variable or exogenous variables. ARCH models are constructed by Engle (1982) and generalized as GARCH by Bollerslev (1986) and Taylor (1986).

Different GARCH models are used to estimate the return volatility of financial instruments. EGARCH (Nelson, 1991), GJR (Glosten, Jagannathan and Runkle;

1993), APARCH (Ding, Granger and Engle; 1993), IGARCH (Engle and Bollerslev; 1986), FIGARCH (Chung, 1999), FIEGARCH (Bollerslev and Mikkelsen, 1996), FIA-PARCH (Tse, 1998) and HYGARCH (Davidson, 2001) are the most known extensions and/or revisions of the ARCH model. The researches show that GARCH models can provide good in-sample parameter estimates and, when the appropriate volatility measure is used, reliable out-of-sample volatility forecasts. Recently the asymmetric normal mixture GARCH model has been used to capture asymmetric volatility in the returns. This paper tests the predictive performance of different GARCH models with normal, Student's t and skewed Student's t distributions of the error terms. Following fifteen models are constructed and compared for estimating return volatility in the Istanbul Stock Exchange.

i) GARCH with normally distributed errors
ii) GARCH with symmetric Student's t distributed errors
iii) GARCH with skewed Student's t distributed errors
iv) GRJ with normally distributed errors
v) GRJ with symmetric Student's t distributed errors
vi) GRJ with skewed Student's t distributed errors
vii) FIGARCH with normally distributed errors
viii) FIGARCH with symmetric Student's t distributed errors
ix) FIGARCH with skewed Student's t distributed errors
x) HYGARCH with normally distributed errors
xi) HYGARCH with symmetric Student's t distributed errors
xii) HYGARCH with skewed Student's t distributed errors
xiii) NM-AGARCH with normally distributed errors
xiv) NM-AGARCH with symmetric Student's t distributed errors
xv) NM-AGARCH with skewed Student's t distributed errors

In a static linear model $\left(y_i = \alpha + \beta\chi_i + \varepsilon_i\right)$, the error term $\left(\varepsilon_i\right)$ is accepted as a random variable with normal distribution and homoscedasticity or equal variance denoted in the Eq. 1.

$$Var\left(\varepsilon_i / \chi_i\right) = E\left[\varepsilon_i - E\left(\varepsilon_i / \chi_i\right)\right]^2 = \sigma^2 \tag{1}$$

Engle (1982) constructs Autoregressive Conditional Heteroscedasticity (ARCH) model to explicit the time-varying variance.

$$\varepsilon_t = z_t \sigma_t$$
$$z_t = i.i.d.D(0.1) \qquad (2)$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

Where D(.) is a probability density function with mean 0 and unit variance. In the Eq. 2, σ_i is the conditional variance of ε_t and varies on time and ω is constant. The conditional variance of ε_t is indeed an increasing function of the square of the shock that occurred in t-1. If ε_{t-1} was large in absolute value, σ_t^2 and ε_t is expected to be large as well (Laurent and Jean-Philippe, 2002).

A high ARCH order has to be selected to catch the dynamics of the conditioal variance. Bollerslev (1986) constructs Generalized ARCH (GARCH) for this issue. The GARCH Model includes the effects of both the linear variance and conditional variance of the past.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{m} \beta_i \sigma_{t-j}^2 \qquad (3)$$

Where again $\{\varepsilon_t\}$ is a sequence of iid random variables with mean 0 and variance $1.0, \alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0$, and $\sum_{i=1}^{max(m.n)} (\alpha_i + \beta_i) < 1$. The latter constrain on $\alpha_i + \beta_i$ implies that the unconditional variance of ε_t is finite, whereas its condtional variance σ_t^2 envolves ever time(Tsay, 2005).

The volatility in the returns increases more than the expected with the negative information if there is asymmetry in the time series. The first GARCH model capturing the asymmetry in the volatility is Exponential GARCH constructed by Nelson (1991).

$$In(\sigma_t) = \delta + (1 + \alpha_l L) f(u_{t-1} / \sigma_{t/1}^{1-2}) + \beta_1 In \sigma_{t-1} \qquad (4)$$

$$f\left(u_{t-1} / \sigma_{t-1}^{1/2}\right) = \theta u_{t-1} + \gamma\left(\left|u_{t-1} / \sigma_{t-1}^{1/2}\right| - E\left|u_{t-1} / \sigma_{t-1}^{1/2}\right|\right) \qquad (5)$$

In the model, the parameters are positive since the logarithmic values of the conditional variance are employed. The Eq. 5 adds the asymmetric characteristic in the model. While "θ" determines the sign of the error term affecting the conditional variance and "γ"states the size effect. If there is asymmetry in the time series, θ should be less than zero.

Gloslen, Jagannathan and Runkle (1993), and Zakoian(1994) state that asymmetry in the return volatility can be modeled by adding a dummy variable into GARCH model. GJR (Threshold GARCH) model is shown on Eq. 6.

$$\sigma_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma 1 u_{t-1}^2 I_{t-1} + \beta_1 \sigma_{t-1} \qquad (6)$$

In the model, if u_{t-1} higher than zero, I_{t-1} is equal to 1, otherwise, equal to zero. ARCH parameters in the conditional variance vary between $\alpha_1 + \gamma 1$ and α_1 in accordance with the sign of the error term. The positive news are affected on the α_1 while the negative news do α_1 and $\gamma 1$. If $\gamma 1$ is higher than 1, it is accepted that there is asymmetry effect while $\gamma 1$ is equal to zero, on the other hand, the news impact curve is symmetric.

In time series with high frequency, the sum of the Alpha and Beta parameters for the conditional variance estimated by GARCH (p,q) model is near or equal to 1 meaning that the volatility effects of the last observations in dataset increase. The same situation is valid for mean equation, as well. When sum of all Auto Regressive (AR) and Moving Average (MA) parameters is equal to 1, Auto Regressive Integrated Moving Average (ARIMA) process is expected (Laurent and Peters, 2002). The GARCH (p,q) process can be modeled as an Auto Regressive Moving Avegare (ARMA) process and written as on the Eq. 7 by using lag operator.

$$\left[1 - \alpha(L) - \beta(L)\right]\varepsilon_t^2 = \omega + \left[1 - \beta(L)\right]\left(\varepsilon_t^2 - \sigma_t^2\right) \qquad (7)$$

The [1-α(L)-β(L)] function has a unit root, the sum of Alpha and Beta parameters is 1 and gives Integrated GARCH model of Engle and Bollerslev (1986). IGARCH model is denoted in the Eq. 8 (Laurent and Peters 2001).

$$\phi(L)(1 - L)\varepsilon_t^2 = \omega + \left[1 - \beta(L)\right]\left(\varepsilon_t^2 - \sigma_t^2\right) \qquad (8)$$

When the IGARCH process is modeled as a conditional variance of the squared error terms, it can be written in GARCH formulation as in Eq. 9.

$$\sigma_t^2 = \frac{\omega}{1 - \beta(L)} + \left\{1 - \phi(L)(1 - L)\left[1 - \beta(L)\right]^{-1}\varepsilon_{t-1}^2 \qquad (9)$$

In the time series, if the fractional difference of yt has a static process, y_t is in the fractional integration. In the $(1\text{-}L)^d = y_t = \varepsilon_t$ equation, if d equals to 0, y_t is static and its autocorrelations are zero. If d is 1, on the other hand, yt has unit root with zero frequency. In case of 0<d<1, the autocorrelations of yt slowly reaches into zero. For that reason, the fractionally integrated models are seen as the models including long memory (Harris and Sollis, 2003). The models with long memory requires in case of high volatility and shocks.

Baillie, Bollerslev and Mikkelsen (1996) constructed Fractionally Integrated GARCH (FIGARCH) model by replacing the lag operator with $(1\text{-L})^d$ in the IGARCH model. FIGARCH-BBM is represented in the Eq. 10.

$$\phi(L)(1-L)^d\left(\varepsilon_t^2-\sigma_t^2\right)=\left[1-\beta(L)\right]\left(\varepsilon_t^2-\sigma_t^2\right)\varepsilon_t^2 \qquad (10)$$

The conditional variance in the FIGARCH (BBM) model is calculated by Eq. 11 where $\omega^*=\left[1-\beta(\mathrm{L})\right]^{-1}, \lambda(\mathrm{L})=\left\{1-\left[1-\beta(\mathrm{L})\right]^{-1}\phi(\mathrm{L})(1-\mathrm{L})^d\right\}\varepsilon_t^2, 0<d<1, \text{and } \sigma_t^2=\omega^*+\lambda(\mathrm{L})$

$$\sigma_t^2=\omega\left[1-\beta(\mathrm{L})\right]^{-1}+\left\{1-\left[1-\beta(\mathrm{L})\right]^{-1}\phi(\mathrm{L})(1-\mathrm{L})^d\right\}\varepsilon_t^2 \qquad (11)$$

Chung (1999) modifies the FIGARCH (BBM) model as it is in the Eq. 12 since ω has theoretical problem and difficulties in the modeling in the practice.

$$\sigma_t^2=\sigma_{t-1}^2+\left\{\left[1-\beta(\mathrm{L})\right]^{-1}\phi(\mathrm{L})(1-\mathrm{L})^d\right\}\left(\varepsilon_t^2-\sigma_{t-1}^2\right) \qquad (12)$$

Where σ^2 is the unconditional variance of ε_t. In this article, FIGARCH model suggested by Chung (1999) is tested.

Anther integrated model developed by Davidson (2002) as a special version of FIGARCH is Hyperbolic GARCH. Davidson (2002) uses near epoch dependency in order to reach long-term memory (Saltoglu, 2003). HYGARCH model can be written in the Eq. 13 (Laurent and Peters, 2002).

$$\sigma_t^2=\omega\left[1-\beta(\mathrm{L})\right]^{-1}+\left\{1-\left[1-\beta(\mathrm{L})\right]^{-1}\phi(\mathrm{L})\left\{1+\alpha\left[(1-\mathrm{L})^d\right]\right\}\right\} \qquad (13)$$

Recently, normal mixture GARCH (NM-GARCH) models have been started to use in detecting the shocks and long-term memory in the returns of the financial instruments. According to Alexander and Lazar (2005), NM-GARCH model can be seen as the Markov switching GARCH model in a restricted form where the transition probabilities are independent of the past state. They argue that the NM-GARCH models are easier to estimate than the Markov switching model constructed by Hamilton and Susmel (1994). What is more, in the NM-GARCH models, the individual variances are only tied with each other through their dependence on the error term.

The methodologies of the NM GARCH models are constructed and formulized by Alexander and Lazar (2005). We follow Alexander and Lazar (2005) in presenting the models.

The asymmetric normal mixture GARCH model has one equation for the mean and K conditional variance components representing different market conditions. The error term has a conditional normal mixture density with zero mean as a weighted average of K normal density functions with different means and variances.

$$\varepsilon_t / I_{t-1} \sim NM\left(p_1,\ldots,p_K,\mu_1,\ldots,\mu_K,\sigma_{1t}^2,\ldots,\sigma_{Kt}^2\right),\ \sum_{i=1}^{K} p_i = 1,\ \sum_{i=1}^{K} p_i\mu_i = 1 \qquad (14)$$

From the Eq. 14, the conditional density of the error term is derived as

$$\eta(\varepsilon_t) = \sum_{i=1}^{K} \mathrm{p}_i\varphi_i \qquad (15)$$

where φ is normal density functions with different constant means μi and different time varying variances σ_{it}^2 for i = 1,…, K.

In the model, it is assumed that K variances follow normal mixture GARCH processes. The NM-GARCH is represented in the Eq. 16.

$$\sigma_{it}^2 = \alpha_0 + \alpha_i\varepsilon_{t-1}^2 + \beta_i\sigma_{it-1}^2 \quad \text{for } i = 1,\ldots,K \qquad (16)$$

NM-AGARCH based on the Engle and Ng, (1993) model is in Eq. 17.

$$\sigma_{\mathrm{it}}^2 = \alpha_0 + \alpha_i\left(\varepsilon_{t-1}^2 - \lambda\right)^2 + \beta_i\sigma_{it-1}^2 \quad \text{for } i = 1,\ldots,K \qquad (17)$$

NM-GJR GARCH based on Glosten et al, (1993) is given by Eq. 18.

$$\sigma_{it}^2 = \alpha_0 + \alpha_i\varepsilon_{t-1}^2 + \lambda_i \mathrm{d}_{t-1}^- \varepsilon_{t-1}^2 + \beta_i\sigma_{it-1}^2 \quad \text{for } i = 1,\ldots,K \qquad (18)$$

where $\mathrm{d}_t^- = 1 \text{ if } \varepsilon_t < 0$, and 0 otherwise.

For both models, the overall conditional variance is

$$\sigma_t^2 = \sum_{i=1}^{K} p_i\sigma_{it}^2 + \sum_{i=1}^{K} p_i\mu_i^2 \qquad (19)$$

When K is bigger than 1, the existence of second, third and fourth moments are assured by imposing less stringent conditions than in the single component in which K is equal to 1. For asymmetric NM-GARCH models, the conditions for the non-negativity of variance and the finiteness of the third moment are represented in the Eq. 20.

$$0 < p_i < 1,\ i = 1,\ldots,K-1,\quad \sum_{i=1}^{K-1} p_i < 1,\ 0 < \alpha_i,\ 0 \le \beta_i < 1 \qquad (20)$$

In the NM-GARCH Model, we should have Eq. 21.

$$m = \sum_{i=1}^{K} p_i \mu_i^2 + \sum_{i=1}^{K} \frac{p_i \omega_i}{(1-\beta_i)} 0, \quad n = \sum_{i=1}^{K} \frac{p_i(1-\alpha_i-\beta_i)}{(1-\beta_i)} > 0 \tag{21}$$

$$\text{and } \omega_i + \alpha_i \frac{m}{n} > 0$$

For the NM-AGARCH model, the Eq. 22 is valid.

$$m = \sum_{i=1}^{K} p_i \mu_i^2 + \sum_{i=1}^{K} \frac{p_i(\omega_i + \alpha_i \lambda_i^2)}{(1-\beta_i)} > 0, n = \sum_{i=1}^{K} \frac{p_i(1-\alpha_i \beta_i)}{(1-\beta_i)} > 0 \tag{22}$$

$$\text{and } \omega_i + \alpha_i \left(\frac{m}{n} + \lambda_i^2 > 0 \right.$$

and for the NM-GRJ GARCH Model, we should have Eq. 23.

$$m = \sum_{i=1}^{K} p_i \mu_i^2 + \sum_{i=1}^{K} \frac{p_i \omega_i}{(1-\beta_i)} > 0, n = \sum_{i=1}^{K} \frac{p_i(1-\alpha_i - 0.5\lambda - \beta_i)}{(1-\beta_i)} > 0 \tag{23}$$

$$\text{and } \omega_i + (\alpha_i + 0.5\lambda) \frac{m}{n} > 0$$

According to Alexander and Lazar (2005), both models have persistent asymmetry, when the conditional returns density is a mixture of normal density components having different means; it is generated by the difference between the expected returns under different market conditions. However, only the NM-AGARCH and NM-GRJ GARCH models have dynamic asymmetry emerging when the λ_i parameters in the component variance processes capture time-varying short-term asymmetries arising from the leverage effect. If λ_i is positive, the conditional variance is higher following a negative unexpected return at time t - 1 than following a positive unexpected return. In the markets, since negative news corresponds to a negative unexpected return, positive λ_i should be expected.

One of the assumptions in linear equation is to estimate the variance with normal distribution. The log-likelihood function of the standard normal distribution is given by Eq. 24(Peters, 2001) where T is the number of observations and $z_t = \frac{\varepsilon_t}{\sigma_t}$. In normal distribution, skewness and kurtosis take the value of (0, 3).

$$L_T = -\frac{1}{2}\sum_{T-1}^{T}\left[In\left(2\pi\right) + In\left(\sigma_t^2\right) + z_t^2\right] \quad (24)$$

Starting with Bollerslev(1987) and Hsieh(1989), Baillie and Bollerslev(1989) and Palm and Vlaar(1997) show that fat-tailed distributions like Student-t perform better to capture higher observed kurtosis. The log-likelihood function of the sdudent-t distribution is given by Eq. 25 (Saltoglu, 2003). Like normal distribution, student-t distribution is also a symmetric.

$$l^{t-dist}\left(\theta\right) = T\left\{In\Gamma\left(\frac{v+1}{1}\right) - In\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}In\left[\pi\left(v-2\right)\right]\right\}$$
$$-\frac{1}{2}\sum_{t=1}^{T}\left(In\left(h_t\right) + \left(1+v\right)In\left(1+\frac{\varepsilon_t^2}{v-2}\right)\right) \quad (25)$$

Where ν is the degree of freedom, $2 < \nu < \infty$ and $\Gamma(.)$ is the gama function. The main drawback of these two distributions is that although student-t may account for fat tails, they are symmetric. Recently, Lambert ve Laurent (2001) applied skewed student-t distribution that is proposed by Fernandez ve Steel (1998) in Value at risk estimation (Peters,2001).

The main advantage of this density is that it considers both asymmetry and fat-tailed-ness(Saltoglu, 2003). If $\Gamma(.)$ denotes the gamma function in the log-likelihood of a standardized skewed student-t is given by Eq. 26(Peters, 2001).

$$l_{skewed-t} = T\left\{In\Gamma\left(\frac{\eta+1}{2}\right) - In\Gamma\left(\frac{\eta}{2}\right) - 0.5In\left[\pi\left(\eta-2\right)\right] + In\left(\frac{2}{\xi+\frac{1}{\xi}}\right) + In\left(s\right)\right\} \quad (26)$$
$$-0.5\sum_{t=1}^{T}\left\{In\sigma_t^3 + \left(1+\eta\right)In\left[1+\frac{\left(S\varepsilon+m\right)^2}{\eta-2}\xi^{-2I_t}\right]\right\}$$

Where $I_t = 1\, if\; z_t \geq -\frac{m}{s}$, $I_t = 0\, if\; z_t < -\frac{m}{s}$, ξ is the asymmetry parameter, η is the degree of freedom of the distribution $m = \frac{\Gamma\left(\frac{\eta+1}{2}\right)\sqrt{\eta-2}}{\sqrt{\pi}\left(\frac{\eta}{2}\right)}\left(\xi - \frac{1}{\xi}\right)$ and

$s = \sqrt{\left(\xi + \frac{1}{\xi^2} - 1\right) - m^2}$ and.

Forecasting ability of GARCH Models has been determined by squared daily returns, Root Mean Squared Error (RMSE) or absolute failure rate that is offered by Basle Committee on Banking Supervision (1996a, 1996b). The Basel backtesting is based on recording daily exceptions as comparing one year of Profit&Loss to a %99 one tail confidence 1 day value at risk with an exception whenever Profit&Loss<-value at risk. Since Basel backtesting procedure do not consider failure rate in shock positions we do not test models with this test. In order to compare asymmetric mixture GARCH and other GARCH models we use two widely used back testing procedures, Kupiec and Christoffersen test.

In Kupiec test, define f as the ratio of the number of observations exceeding Var(x) to the number of total observation (T) and pre-specified VaR level as α (Tang and Shieh, 2006). The statistic of Kupiec LR test is given by Eq. 27 (Kupiec, 1995). LR is distributed as chi-square distribution.

$$LR = 2\left\{ log\left[f^{x}\left(1-f\right)^{T-x} \right] - log\left[\alpha^{x}\left(1-\alpha\right)^{T-x} \right] \right\} \quad (27)$$

The VaRs of α quantile for long and short trading position are computed as in Equation 28, 29 and 30 for normal, student-t and skewed student-t respectively(Tang and Shieh, 2006).

$$VAR_{long} = \hat{\mu}_t - z_{\alpha}\hat{\sigma}_t, \qquad VAR_{short} = \hat{\mu}_t - z_{\alpha}\hat{\sigma}_t \quad (28)$$

$$VAR_{long} = \hat{\mu}_t - st_{\alpha,\nu}\hat{\sigma}_t, \qquad VAR_{short} = \hat{\mu}_t - st_{\alpha,\nu}\hat{\sigma}_t \quad (29)$$

$$VAR_{long} = \hat{\mu}_t - skst_{\alpha,\nu,\xi}\hat{\sigma}_t, \qquad VAR_{short} = \hat{\mu}_t - skst_{\alpha,\nu,\xi}\hat{\sigma}_t \quad (30)$$

Where z_{α}, $st_{\alpha,\nu}$ and $skst_{\alpha,\nu,\xi}$ are left or right tail quantile at α % for normal, student-t and skewed student-t distributions respectively.

Christoffersen test (Christoffersen, 1998) focuses on the probability of failure rate and is based on testing whether $\Pr(r_1 < v_t) = p$ after conditioning on all information available at time t (Sarma et all, 2001). The importance of testing conditional coverage arises with volatility clustering in financial time series.

Christoffersen test can be applied as follows (Saltoglu, 2003). Define P^{α} = $\Pr(y_t < VaR_t(\alpha))$ to test $H_0: p^{\alpha} = \alpha$ against $H_1: p^{\alpha} \neq \alpha$. Consider $\{1(y_t < VaR(\alpha))\}$ which has a binomial likelihood $L(p^{\alpha}) = (1-p^{\alpha})^{n_0}(p^{\alpha})^{n_1}$. where $n_0 = \sum_{t=R}^{T} 1(y_t > VaR_t$

$(\alpha))$ and $n_1 = \sum_{t=R}^{T} 1(y_t < VaR_t(\alpha))$. Under the null hypothesis, it becomes . L(α) = (1 - α)$^{n_0}\alpha^{n_1}$. Thus the likelihood ratio test statistics is in Eq. 31.

$$LR = 2In\left(L(\alpha)\right) / L(p) \xrightarrow{d} \chi(1) \quad (31)$$

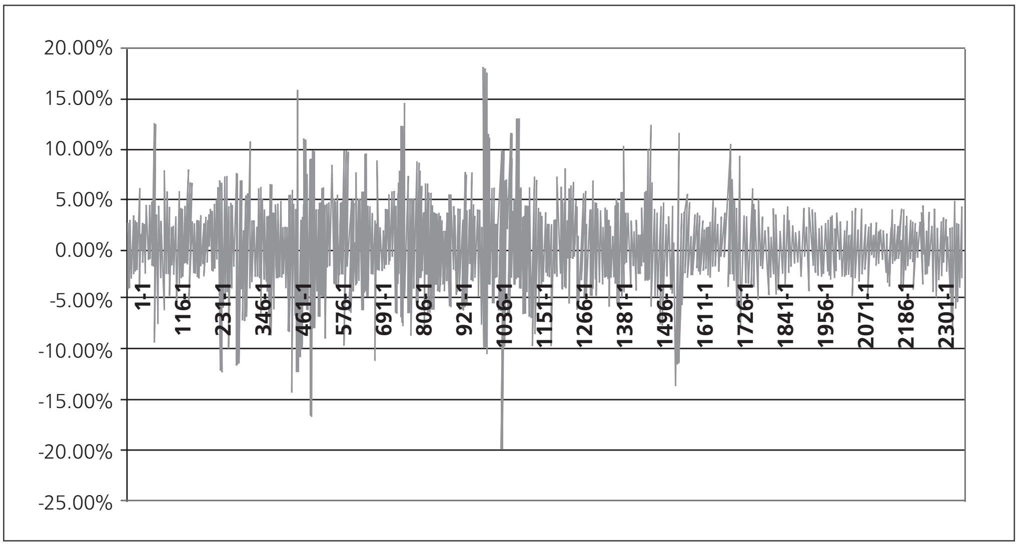
We estimate VaR with α = 0.01 and α = 0.05 confidence interval and backtest VaR models with Kupiec in-sample and out-of-sample forecasting and Christoffersen in-sample and out-of-sample forecasting test. We chose %99 C.I. as Basel II requires %99 C.I. and %95 C.I. to compare VaR results with different C.I. level.

4. Data and Empirical Results

Data

Istanbul Stock Exchange Rate (ISE-100 Index) is from Bloomberg. Our dataset covers 2412 daily observations from 01/10/1996 to 11/07/2006. We constituted the series in log-differenced level. Figure 1 shows ISE Index in log-differenced series. By performing Augmented Dickey–Fuller (Dickey and Fuller, 1981) test we found that ISE Index is stationary at log differenced level (as Augmented Dickey-Fuller test of I(1) with 0 lags is equal to -48.2929 {<%1}). The estimation process is run using 10 years of data (1996-2005) while the remaining 5 year (252*5 days) is used for out-of-sample forecasting.

Figure 1. ISE Log-differenced series



Empirical Results

In this subsection, we report estimation and Kupiec and Christoffersen tests results for Asymmetric Normal Mixture GARCH and other GARCH Models and detailed in Methodology section. We used Ox programming language (see Doornik, 1999) and parameters are estimated using Quasi Maximum Likelihood technique (Bollerslev and Woolridge, 1992) and BFGS quasi-Newton method optimization algorithm used. Estimation of Asymmetric Normal Mixture GARCH is performed with modified version of Alexander and Lazar(2006) codes and other GARCH models is carried out with G@rch 3.0 (Laurent and Peters, 2002).

Table 1 and Table 2 shows GARCH, GRJ, FIGARCH and HYGARCH estimation results with normal, student-t and skewed student-t distributions. α and β_1 parameters for all of the models statistically significant. Student parameters (ν) are statistically significant for all the GARCH models and thus shows that time series is fat tailed. For the skewed student-t distribution, the asymmetric parameters (ξ) are negative and statistically significant for all GARCH models. Thus show that the density distribution of ISE skewed to to left.

Table 1. Estimation Results from GARCH(1,1) and GRJ(1,1)

	GARCH	GARCH-t	GARCH-Skew	GJR	GJR-t	GJR-Skew
ω	0.139** (4.52)	0.169** (3.554)	0.181** (3.62)	0.151** (4.74)	0.197** (3.827)	0.204** (3.850)
α	0.110** (12.26)	0.109** (7.48)	0.115** (7.49)	0.099** (10.43)	0.088** (5.815)	0.0936** (5.842)
β_1	0.880** (98.00)	0.876** (59.21)	0.871** (56.45)	0.876** (95.54)	0.867** (55.65)	0.8650** (53.90)
ν -Student t	-	6.560** (7.88)	-	-	6.490** (8.032)	-
ξ -Skewness	-	-	-0.056* (-2.05)	-	-	-0.0479* (-1.736)
ν - Skewness	-	-	6.508** (7.62)	-	-	6.4409** (7.779)
$\gamma 1$ - GJR	-	-	-	0.0302** (2.50)	6.490** (8.032)	0.0559** (2.468)
Volatility	0.0400219	0.0352803	0.0368966	0.0251961	0.0214036	0.0222843
LogLike	5245.17	5302.81	5304.89	5247.10	5306.9241	5308.3969
	-4.375	-4.423	-4.423	-4.376	-4.425	-4.42604
AIC	-4.375	-4.423	-4.423	-4.376	-4.425	-4.42604

Table 2. Estimation Results from FIGARCH(1,d,1) and HYGARCH(1,d,1)

	FiGARCH Chung	FiGARCH Chung-t	FiGARCH Chung-Skew	HyGARCH	HyGARCH-t	HyGARCH-Skew
ω	8.638** (4.106)	8.346** (2.637)	9.051** (2.676)	1.081** (2.623)	1.5037* (2.48)	1.5628** (2.586)
α	0.2948** (4.468)	0.2734* (2.12)	0.2726** (2.09)	-0.5278 (-1.42)	-0.646* (-1.740)	-0.6725* (-1.959)
β_1	0.5216** (7.596)	0.466** (3.33)	0.4630** (3.26)	-0.4924 (-1.27)	-0.610 (-1.550)	-0.6379 (-1.746)*
ν -Student t	-	6.985** (8.00)	-	-	7.070** (7.52)	-
ξ -Skewness	-	-	-0.0631* (-2.25)	-	-	-0.061** (-2.19)
ν -Skewness	-	-	6.988** (7.714)	-	-	7.021** (7.257)
d FiGARCH	0.3958** (10.66)	0.3679** (6.56)	0.3698** (6.64)	0.0403 (0.969)	0.0654 (0.982)	0.0703 (1.073)
HyGARCH In(α)	-	-	-	1.5988* (1.71)	1.138 (1.314)	1.0852 (1.383)
Volatlity	0.068605	0.0566118	0.0585214	0.007316	0.008161	0.008224
LogLike	5264.35	5316.50	5319.08	5267.16	5317.44	5319.8413
AIC	-4.390	-4.433	-4.43496	-4.39246	-4.43359	-4.43476

Estimated long memory parameter of d for FiGARCH and hyperbolic parameter of In(α) for HyGARCH are statistically significant (Table 2).

As reported in Table 3, ω, α, β_1 and normal mixture γ (Gamma) parameter statistically significant for all of the Asymmetric Normal Mixture Models. Besides student-t and skewed student-t parameters ν-Student t, ξ-Skewness and ν-Skewness are also statistically significant. These results shows that Asymmetric Normal Mixture GARCH models may perform better and this hypothesis can be tested with backtesting procedures such as Kupiec and Christoffersen tests.

Table 4 shows Root Mean Squared Errors(RMSE), Mean Squared Errors(MSE), information criteria test and Nyblom test(Nyblom, 1994) results. Nyblom tests statistics shows that all of the models' parameters are stable.

Table 3. Estimation Results from NORMAL MIXTURE-AGARCH(1,1)

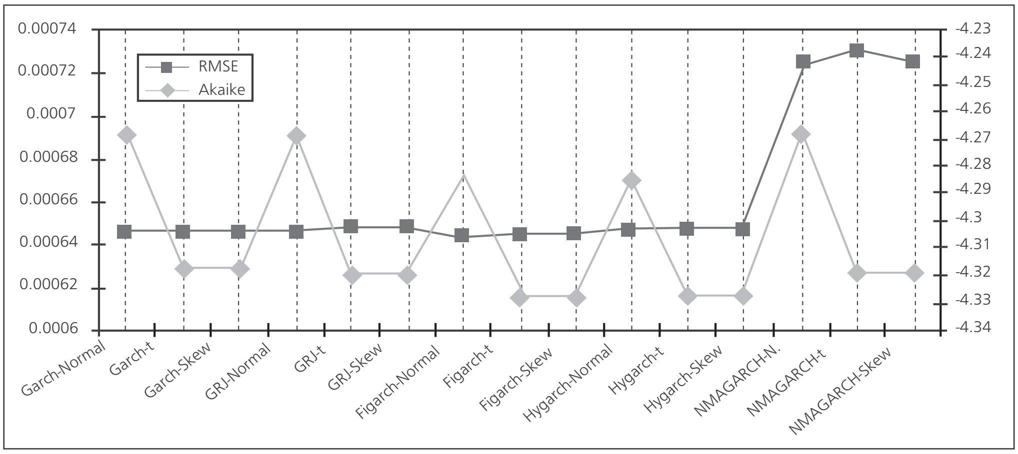
	NM-AGARCH	NM-AGARCH-t	NM-AGARCH-Skew
ω	0.155711** (4.822)	0.190152 ** (3.661)	0.199841 ** (3.732)
α	0.116320 ** (12.26)	0.119105 ** (7.554)	0.122624 ** (7.542)
β_1	0.873762 ** (92.82)	0.863852 ** (54.26)	0.861064** (52.75)
ν -Student t	-	6.469056 ** (8.071)	-
ξ -Skewness	-	-	-0.047852 * (-1.734)
ν -Skewness	-	-	6.420465 ** (7.820)
γ -Normal Mixture	0.002845 ** (2.743)	0.005639 ** (3.052)	0.005237 ** (2.825)
Volatility	0.0408031	0.036578	0.0378326
LogLike	5247.2477	5307.6638	5309.1297
AIC	-4.37667	-4.42626	-4.42665

Table 4. Forecast Evaluation Measures*

Method	MSE	RMSE	Akaike	$Q^2(10)$**	Nyblom test
GARCH-Normal	4.16e-007	0.000645	-4.268528	17.639 [0.0241018]	1.76915
GARCH-t	4.165e-007	0.0006454	-4.317390	17.7709 [0.0230116]	2.12799
GARCH-Skew	4.169e-007	0.0006457	-4.317207	17.437 [0.0258668]	2.50175
GRJ-Normal	4.168e-007	0.0006456	-4.269172	15.9615 [0.042934]	2.0413
GRJ-t	4.192e-007	0.0006475	-4.320163	14.832 [0.0624966]	2.56872
GRJ -Skew	4.195e-007	0.0006477	-4.319635	14.7126 [0.0649816]	2.9462
FiGARCH-Normal	4.14e-007	0.0006434	-4.282631	18.8561 [0.0156486]	0.974128
FiGARCH-t	4.147e-007	0.000644	-4.327248	19.8371 [0.0109701]	1.04441
FiGARCH-Skew	4.151e-007	0.0006443	-4.327241	19.9293 [0.0106068]	1.31536
HyGARCH-Normal	4.165e-007	0.0006454	-4.284598	16.6122 [0.0344101]	1.27277
HyGARCH-t	4.175e-007	0.0006461	-4.327595	17.1764 [0.0283242]	1.18444
HyGARCH-Skew	4.176e-007	0.0006462	-4.327535	17.1236 [0.0288478]	1.46686
NMAGARCH-N .	5.239e-007	0.0007238	-4.268447	16.9429 [0.0307096]	1.98177
NMAGARCH-t	5.240e-007	0.0007301	-4.319692	16.5072 [0.0356698]	2.62355
NMAGARCH-Skew	5.239e-007	0.0007238	-4.319203	16.3212 [0.0380071]	2.99794

* *1 day ahead out-of-sample forecasting based on 252 days evaluation.*
** *Q-Statistics on Squared Standardized Residuals*

Figure 2. RMSE and Akaike Values



As reported in Çifter (2004), RMSE or MSE may not be adequate backtesting test as these tests do not consider tail probability and overshooting effects. This can be seen in Figure 2 as RMSE is maximum for NMGARCH models where Akaike criteria tests are not maximum for NMGARCH models. Kupiec and Christoffersen tests can be more consistent to compare GARCH models.

We compared VaR models with Kupiec test for long and short trading positions. We define a failure rate for long trading position as percentage of negative returns smaller than one-step ahead VaR for long position (left tail of the density distribution of the returns) and a failure rate as the percentage of positive returns larger than one-step ahead VaR for short position (right tail of the density distribution of the returns).

The empirical results based on Kupiec in-sample forecasting test are summarized in Table 5 and Figure 3. The table contains Kupiec failure rates for short and long position VaR. Number of in-sample-forecasting is 15 days and confidence interval(C.I.) is chosen as with α=0.01 and α=0.05. Table 5 can be read as follow. If the model is estimated accurately, it should explain the actual observations very well. The failure rate should be equal to the pre-specified VaR level, and Kupiec LR test would not reject its null hypothesis as failure rate equals to α (Tang and Shieh, 2006).

In sample VaR results for long and short trading positions are reported in Table 5. The empirical results of Kupiec in-sample forecasting test shows that NMAGARCH with Gaussian distribution for short position and FIGARCH(1,d,1) with skewed student-t distribution performs better for α=0.05 where NMAGARCH with student-t for short position and GRJ with student-t and HYGARCH with skewed student-t distribution for long position performs better for α=0.01. These results show that none of the model outperforms other models based on Kupiec in-sample forecasting.

Since in-sample forecasting estimates VaR with only know the past performance, out-of-sample forecasting is more consistent. Our out-of-sample forecast evaluation uses one step ahead prediction for 252x5 days forecast sample. Out of sample VaR results for long and short trading positions are reported in Table 6 and Figure 4. The empirical results of Kupiec out-of-sample forecasting test shows that FIGARCH(1,d,1) with skewed student-t distribution and HYGARCH(1,d,1) with skewed student-t distribution for short position and HYGARCH(1,d,1) with student-t distribution performs better for α=0.05 where NMAGARCH with Gaussian distribution for short position and GRJ with Gaussian distribution and NMAGARCH with Gaussian distribution for long position performs better for α=0.01. The empirical evidence is in favor of the FIGARCH with skewed student-t, HYGARCH with skewed student-t, GRJ with Gaussian, GRJ with student-t and NMAGARCH with Gaussian distribution based on Kupiec in-sample and out-of-sample forecasting.

Christoffersen test VaR results for in-sample and out-of-sample forecasting are reported in Table 7 and Figure 5. The empirical results in-of-sample forecasting results shows that NM-AGARCH with Gaussian distribution for α=0.05 and FIGARCH(1,d,1) with Gaussian distribution for α=0.01 performs better where out-of-sample forecasting results show that GARCH(1,1) with Gaussian distribution for α=0.05 and NMAGARCH with student-t for α=0.01 performs better.

Empirical results based on Kupiec and Christoffersen tests show that volatility model should be chosen in accordance with confidence interval and trading positions. However, NMAGARCH model has better predictive performance for higher confidence interval. The Basel II Accord requires accurate volatility model, which is statistically significant at 99 % confidence level.

Figure 6 shows out-of-sample estimation for GARCH and NM-AGARCH with gaussian and skewed student-t distribution. NM-AGARCH captures fat-tailed behavior of the data(shocks) better than GARCH.

Table 5. In-Sample-Forecasting Kupiec Test

In Sample Forecasting %95 Confidence Interval$^{\delta}$						
	VaR for Short position			VaR for Long position		
	Failure Rate	Kupiec LR	p-value	Failure Rate	Kupiec LR	p-value
GARCH-Normal	0.94894	0.02651	0.87066	0.043134	1.1801	0.27734
GARCH-t	0.94366	0.92453	0.33629	0.046655	0.27346	0.60102
GARCH-Skew	0.94190	1.4943	0.22155	0.044014	0.89142	0.34509
GRJ-Normal	0.94630	0.31955	0.57188	0.039613	2.7699	0.09605**
GRJ-t	0.94190	1.4943	0.22155	0.040493	2.3054	0.12893*
GRJ-Skew	0.94014	2.1927	0.13866*	0.038732	3.2807	0.07009**
FiGARCH-Normal	0.94718	0.18649	0.66586	0.046655	0.27346	0.60102
FiGARCH-t	0.93662	3.9618	0.04654**	0.047535	0.14761	0.70083
FiGARCHSkew	0.93926	2.5891	0.10760*	0.048415	0.060655	0.80546
HyGARCH - Normal	0.94718	0.18649	0.66586	0.046655	0.27346	0.60102
HyGARCH-t	0.94454	0.68909	0.4064	0.047535	0.14761	0.70083
HyGARCH-Skew	0.94102	1.8277	0.17640	0.045775	0.43889	0.50766
NMAGARCH-No.	0.95033	0.0056353	0.94016	0.037980	7.9194	0.004890*
NMAGARCH-t	0.94783	0.23440	0.62828	0.041319	4.0301	0.04469**
NMAGARCH-Skew	0.94491	1.2678	0.26019	0.038815	6.8146	0.00904**
In Sample Forecasting %99 Confidence Interval$^{\delta}$						
	VaR for Short position			VaR for Long position		
	Failure Rate	Kupiec LR	p-value	Failure Rate	Kupiec LR	p-value
GARCH-Normal	0.98680	1.0703	0.30087	0.017606	5.4119	0.02000**
GARCH-t	0.99032	0.011646	0.91406	0.010563	0.035762	0.85001
GARCH-Skew	0.98856	0.22852	0.63263	0.010563	0.035762	0.85001
GRJ-Normal	0.98504	2.4542	0.11721*	0.018486	6.6087	0.01014**
GRJ-t	0.99120	0.17138	0.67889	0.0096831	0.011646	0.91406
GRJ-Skew	0.99120	0.17138	0.67889	0.0079225	0.53322	0.46526
FiGARCH-Normal	0.98327	4.3170	0.03773**	0.018486	6.6087	0.01014**
FiGARCH-t	0.98944	0.035762	0.85001	0.010563	0.035762	0.85001
FiGARCHSkew	0.98856	0.22852	0.63263	0.011444	0.22852	0.63263
HyGARCH - Normal	0.98327	4.3170	0.03773**	0.017606	5.4119	0.02000**
HyGARCH-t	0.99120	0.17138	0.67889	0.011444	0.22852	0.63263
HyGARCH-Skew	0.98944	0.035762	0.85001	0.0096831	0.011646	0.91406
NMAGARCH-No.	0.98539	4.4988	0.03391**	0.013356	2.4657	0.11636
NMAGARCH-t	0.98998	6.7415e-5	0.99345	0.0095993	0.039377	0.84270
NMAGARCH-Skew	0.98790	1.0035	0.31647	0.0087646	0.38543	0.53471

**, ** are %5 and %10 confidence level respectively.*

δ Number of forecast:15 days

Table 6. Out-of-Sample Forecasting Kupiec Test

Out-of-Sample Forecasting %95 Confidence Interval[δ]						
	VaR for Short position			VaR for Long position		
	Failure Rate	Kupiec LR	p-value	Failure Rate	Kupiec LR	p-value
GARCH-Normal	0.97222	15.505	8e-005**	0.025397	19.442	1e-005**
GARCH-t	0.97222	15.505	8e-005**	0.027778	15.505	8e-005**
GARCH-Skew	0.96905	11.071	0.00087**	0.026190	18.068	2e-005**
GRJ-Normal	0.97143	14.312	0.00015**	0.026190	18.068	2e-005**
GRJ-t	0.96905	11.071	0.00087**	0.028571	14.312	0.00015**
GRJ-Skew	0.96905	11.071	0.00087**	0.027778	15.505	8e-005**
FiGARCH-Normal	0.96825	10.099	0.00148**	0.029365	13.177	0.00028**
FiGARCH-t	0.96429	5.9868	0.014413*	0.033333	8.3072	0.00394**
FiGARCHSkew	0.95873	2.1441	0.14312	0.032540	9.1778	0.00244**
HyGARCH - Normal	0.96508	6.7128	0.009572*	0.031746	10.099	0.00148**
HyGARCH-t	0.96032	3.0295	0.081763*	0.035714	5.9868	0.014413*
HyGARCH-Skew	0.95873	2.1441	0.14312	0.030159	12.097	0.00050**
NMAGARCH-No.	0.97222	15.505	8.22-e5**	0.027778	15.505	8.22-e5**
NMAGARCH-t	0.97063	13.177	0.00026**	0.027778	15.505	8.22e-5**
NMAGARCH-Skew	0.96984	12.097	0.0005 **	0.028571	14.312	0.00015**

Out-of-Sample Forecasting %99 Confidence Interval[δ]						
	VaR for Short position			VaR for Long position		
	Failure Rate	Kupiec LR	p-value	Failure Rate	Kupiec LR	p-value
GARCH-Normal	0.99365	1.9489	0.16271	0.007936	0.58318	0.44507
GARCH-t	0.99683	8.0799	0.00447**	0.0055556	2.9961	0.083466*
GARCH-Skew	0.99603	6.0036	0.01427**	0.0055556	2.9961	0.083466*
GRJ-Normal	0.99444	2.9961	0.08346**	0.0079365	0.0079365	0.44507
GRJ-t	0.99603	6.0036	0.01427**	0.0055556	2.9961	0.083466*
GRJ-Skew	0.99603	6.0036	0.01427**	0.0055556	2.9961	0.083466*
FiGARCH-Normal	0.99444	2.9961	0.083466*	0.0095238	0.029325	0.86403
FiGARCH-t	0.99524	4.3316	0.037411*	0.0087302	0.21442	0.64333
FiGARCHSkew	0.99444	2.9961	0.083466*	0.0071429	1.1539	0.28274
HyGARCH - Normal	0.99365	1.9489	0.16271	0.011111	0.15167	0.69695
HyGARCH-t	0.99444	2.9961	0.083466*	0.0063492	1.9489	0.16271
HyGARCH-Skew	0.99444	2.9961	0.083466*	0.0063492	1.9489	0.16271
NMAGARCH-No.	0.99286	1.1539	0.28274	0.0079365	0.58318	0.44507
NMAGARCH-t	0.99603	6.0036	0.01427**	0.0055556	2.9961	0.083466*
NMAGARCH-Skew	0.99603	6.0036	0.0142 **	0.0047619	4.3316	0.03741 *

**, ** are %5 and %10 confidence level respectively.*

*δ Number of forecast:252*5 days and 1 day ahead*

Table 7. Christoffersen Test

In Sample Forecasting at %95.00 Confidence Interval&

Method	LR	p-value	Method	LR	p-value
GARCH-Normal	5.7993	0.016032	FiGARCH-Skew	2.0046	0.72034
GARCH-t	4.0301	0.044694	HyGARCH-Normal	3.6410	0.056372
GARCH-Skew	1.4909	0.22207	HyGARCH-t	1.2635	0.26099
GRJ-Normal	7.9194	0.0048908	HyGARCH-Skew	0.00563	0.94016
GRJ-t	4.0301	0.044694	NMAGARCH-No.	7.98953	0.003214
GRJ-Skew	2.2914	0.13009	NMAGARCH-t	4.157480	0.039824
FiGARCH-Normal	5.7993	0.016032	NMAGARCH-Skew	5.93530	0.087203
FiGARCH-t	2.0047	0.15681			

In Sample Forecasting at %99.00 Confidence Interval&

Method	LR	p-value	Method	LR	p-value
GARCH-Normal	2.4657	0.11636	FiGARCH-Skew	1.4241	0.23273
GARCH-t	0.37426	0.54069	HyGARCH-Normal	1.4241	0.021389
GARCH-Skew	1.4241	0.23273	HyGARCH-t	0.65273	0.41914
GRJ-Normal	2.4657	0.11636	HyGARCH-Skew	1.9122	0.16672
GRJ-t	6.7415e-5	0.99345	NMAGARCH-No.	2.4952	0.09250
GRJ-Skew	0.37426	0.54069	NMAGARCH-t	0.039377	0.84272
FiGARCH-Normal	6.1472	0.013162	NMAGARCH-Skew	0.170710	0.67948
FiGARCH-t	0.17071	0.67948			

Out-of-Sample Forecasting at %95.00 Confidence Interval$^{\delta}$

Method	LR	p-value	Method	LR	p-value
GARCH-Normal	19.442	1.0368e-005	FiGARCH-Skew	3.0295	0.081763
GARCH-t	15.505	8.2294e-005	HyGARCH-Normal	10.099	0.0014837
GARCH-Skew	12.097	0.00050507	HyGARCH-t	5.9865	0.014413
GRJ-Normal	18.068	2.1312e-005	HyGARCH-Skew	4.0816	0.043354
GRJ-t	14.312	0.00015486	NMAGARCH-No.	15.505	8.2294e-005
GRJ-Skew	14.312	0.00015486	NMAGARCH-t	15.507	8.2296e-005
FiGARCH-Normal	13.177	0.00028346	NMAGARCH-Skew	14.312	0.00015489
FiGARCH-t	8.3072	0.0039488			

Out-of-Sample Forecasting at %99.00 Confidence Interval$^{\delta}$

Method	LR	p-value	Method	LR	p-value
GARCH-Normal	0.58318	0.4451	FiGARCH-Skew	0.21442	0.64333
GARCH-t	2.9961	0.083466	HyGARCH-Normal	0.15167	0.69695
GARCH-Skew	1.1539	0.28274	HyGARCH-t	1.9489	0.16271
GRJ-Normal	0.58318	0.44507	HyGARCH-Skew	0.21442	0.64333
GRJ-t	2.9961	0.083466	NMAGARCH-No.	0.68250	0.23510
GRJ-Skew	2.9961	0.083466	NMAGARCH-t	3.12450	0.051542
FiGARCH-Normal	0.029325	0.86403	NMAGARCH-Skew	2.98457	0.085287
FiGARCH-t	0.21442	0.64333			

& *Number of forecast(in-sample):15 days ahead*

$^{\delta}$ *Number of forecast(out-of-sample): 1 day ahead for 252*5 days sample*

Figure 3. In-sample Kupiec test p-value

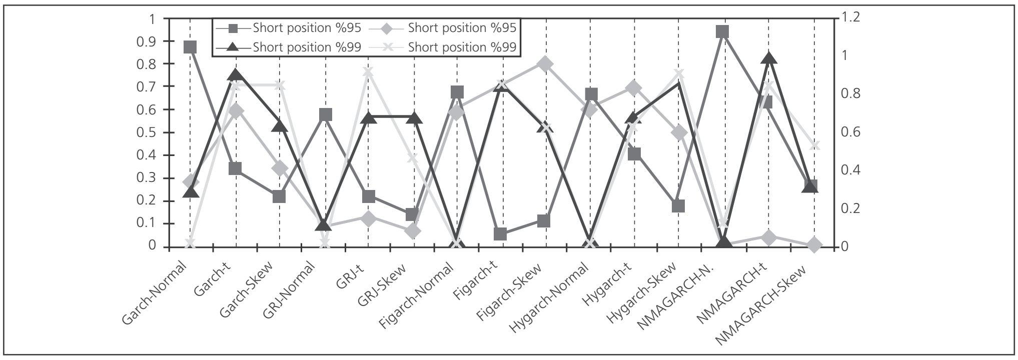


Figure 4. Out-of-sample Kupiec test p-value

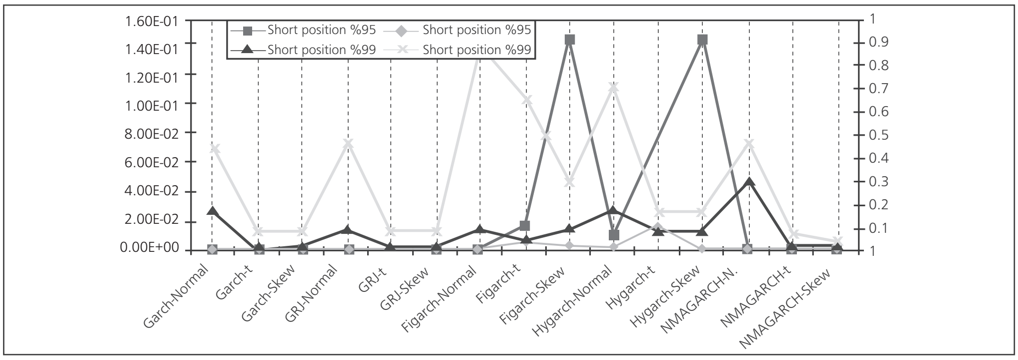


Figure 5. Christoffersen Test p-value

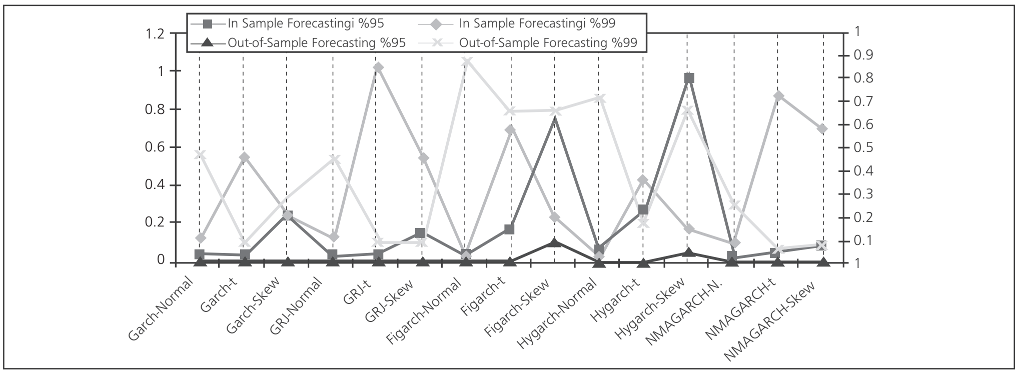
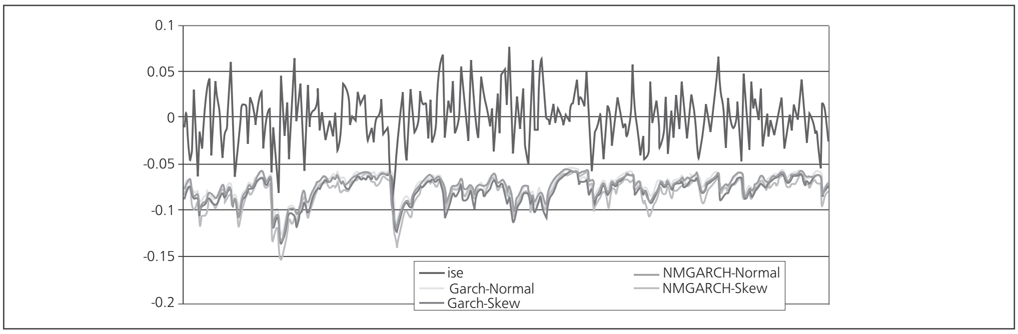


Figure 6. Out-of-Sample Forecasting (Last 252 days)



5. Conclusion

Though volatility in stock returns provides opportunity in earning profit for traders, it is a threat for risk managers in balancing risk-return relationship. In emerging markets, return volatility is relatively high due to low market volume, unstable political and economic conditions, and hot money from international investment portfolios. High volatility and non-linear returns in stock prices require advanced volatility measurement models based on non-normal distribution of returns. They should catch the fat tails and regime switches, which are not easy to be estimated and modeled with static econometric models.

In this paper, the return volatility of stocks traded in the Istanbul Stock Exchange is estimated by different GARCH models. The research is especially interested in the predictive performance of Asymmetric Normal Mixture GARCH (NMAGARCH) based on Kupiec and Christoffersen tests for the Istanbul Stock Exchange National 100 Index. In this respect, this article includes the first research employing the NMAGARCH model in Turkish equity markets. What is more, it has original contribution to the finance literature by conducting reality check of the NMAGARCH model with comparing the classical GARCH models.

By examining fifteen GARCH models with alternative return distribution assumptions, the paper shows that the NMAGARCH perform better based on 99 %confidence interval out-of-sample forecasting Christoffersen test. On the other hand, FiGARCH with skewed student-t, HyGARCH with skewed student-t, GRJ with normal, GRJ with student-t and NMAGARCH with Gaussian distribution perform better based on 95 % confidence interval out-of-sample forecasting Christoffersen test and Kupiec tests.

The empirical evidence has a crucial concluding remark in prediction of stock market volatility. These results show that none of the model including NMAGARCH outperforms other models in all cases as trading position or confidence intervals and the real implications of these results for Value-at-Risk estimation is that volatility model should be chosen according to confidence interval and trading positions. However, NMAGARCH model has better predictive performance for higher confidence interval. The Basel II Accord requires accurate volatility model, which is statistically significant at 99 % confidence level. The paper show that for accurate internal volatility models being proper for the Basel II Accord, advanced models based on financial computing should be constructed by examining the nature of the markets under investigation.

References

1. Ackert, L. F., and Racine, M. D. (1999). Time Varying Volatility in Canadian and US Stock Index and Index Futures Markets: A Multivariate Analysis, *Federal Reserve Bank of Atlanta Working Paper Series,* No: 98-14.
2. Alexander, C. and Lazar, E. (2003). Symmetric Normal Mixture GARCH, *ISMA Center Discussion Paper in Finance,* No:9.
3. Alexander, C. and Lazar, E. (2005). The Equity Index Skew, Market Crashes and Asymmetric Normal *Mixture GARCH, ISMA Center, Mimeo*
4. Alexander, C. and Lazar, E. (2006). Normal Mixture GARCH(1,1):Applications to Exchange Rate Modeling, *Journal of Applied Econometrics,* 21(3): 307-336.
5. Andersen, T.G. and Bollerslev, T. (1998). DM-Dollar Volatility: Intraday Activity Patterns, Macroeconomic Announcements, and Longer-Run Dependencies, *Journal of Finance,* 53(1):.219-265.
6. Baillie, R. T., Bollerslev, T., and Mikkelsen, H.O. (1996). Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics,* 74: 3-30.
7. Baillie, R. T. and Bollerslev, T. (1989). The Message in Daily Exchange Rates: A Conditional-Variance Tale, *Journal of Business and Economic Statistics,* 7:297-305.
8. Basle Committee on Banking Supervision. (1996a). Amendment to the Capital Accord to Incorporate Market Risks, *Basle, Switzerland:* BIS.
9. Basle Committee on Banking Supervision. (1996b). Supervisory Framework for the Use of 'Backtesting' in Conjunction with the Internal Models Approach to Market Risk Capital Requirements. *Manuscript, Basle, Switzerland: BIS.*
10. Bates, D. S. (2003). Empirical Options Pricing: A Retrospection, *The Journal of Econometrics,* 116: 387-404.
11. Bates, D. S. (1991). The Crash of '87: Was It Expected? The Evidence from Options Markets, *Journal of Finance,* 46: 1009-1044.
12. Bekaert, G., and Wu, G. (2000). Asymmetric Volatility and Risk Equity Markets, *The Review of Financial Studies,* 13(1): 1-42.
13. Bollerslev, T. and Woolridge, J. M. (1992). Quasi-maximum Likelihood Estmation Inference in Dynamic Models with Time-varying Covariances, *Econometric Theory,* 11: 143-172.

14. Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics*, 31: 307–327.

15. Bollerslev, T., and Ghysels, E. (1996). Periodic Autoregressive Conditional Heteroskedasticity, *Journal of Business and Economics Statistics,* 14: 139–152.

16. Bollerslev, T., Chou, R. Y. and Kroner, K. F. (1992). ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence, *Journal of Economics and Statistics,* 69: 542-547.

17. Bollerslev, T. (1987). A Conditional Heteroskedasticity Time Series Model for Speculative Prices and Rates of Return, *Review of Economic and Statistics.* 69: 542-547.

18. Bollerslev, T., and Mikkelsen, H. O. (1996). Modelling and pricing long memory in stock market volatility, *Journal of Econometrics*, 73: 151-84.

19. Çifter, A. (2004). Asymmetric and Fractionally Integrated GARCH Models with (Skewed) Student-t and Ged Distribution in Risk Management: An Application on Eurobond, Presented in *VIII. National Finance Symposium, Istanbul Technical University (in Turkish).*

20. Christoffersen, P. F. (1998). Evaluating Interval Forecasts, *International Economic Review,* (39): 841-862.

21. Christoffersen, P. F., and Jacobs, Kris. (2004). Which GARCH Model for Option Valuation?, *Management Science*, 50: 1204-1221.

22. Christoffersen, P. F., Heston, S., and Jacobs, K. (2004). Option Valuation with Conditional Skewness. Fortcoming in The *Journal of Econometrics.*

23. Cung, C.-F. (1999). Estimating the Fractionally Integrated GARCH Model, *National Taiwan University, Working Paper.*

24. Darrat, A., and Benkato, O. (2003). Interdependence and Volatility Spillovers under Market Liberalization: The case of Istanbul Stock *Exchange, Journal of Business, Finance & Accounting,* 30:1089-1114.

25. Ding, Z., Granger, C. W. J. and Engle, R. F. (1993). A Long Memory Property of Stock Market Returns and a New Model, *Journal of Empirical Finance,* 1: 83–106.

26. Dickey, D. A., and Fuller, W. A. (1981). Likelihood ratio statistics for autoregressive time series with a unit root, *Econometrica*, 49: 1057–1072.

27. Davidson, J. (2001). Moment and Memory Properties of Linear Conditional He-

teroskedasticity Models, *Manuscript, Cardiff University.*

28. Davidson, J. (2002). Moment and Memory Properties of Linear Conditional Heteroscedasticity Models, Working Paper, http://www.cf.ac.uk/carbs/econ/davidsonje.
29. Doornik, J.A. (1999). *An Object Oriented Programming Language*, UK: Timberlake Consultant, Third Ed.
30. Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimate of the Variance of United Kingdom Inflation, *Econometrica*, 50: 987-1007.
31. Engle, R. F. and Tim, B. (1986). Modeling the Persistence of Conditional Variances, *Econometric Reviews,* 5: 1-50.
32. Engle, R. F. and Ng, V. K. (1993). Measuring and Testing the Impact of News on Volatility, *Journal of Finance*, 48: 1749-1778.
33. Fernandez, C., and Stell, M. (1998). On Bayesian Modeling of fat tails and skewness, *Journal of the American Statistical Association*, 93: 359-371.
34. Glosten, L. R., Jagahannathan, R., and Runkle, D. E. (1993). On the Relationship between the Expected Value and The Volatility of the Nominal Excess Return on Stocks, *Journal of Finance,* 48: 1779-1801.
35. Hamilton, J. D., and Susmel R. (1994). Autoregressive Conditional Het-eroskedasticity and Changes in Regime, *Journal of Econometrics,* 64: 307-333.
36. Harris, R. and Sollis, R. (2003). *Applied Time Series Modeling and Forecasting,* UK: Wiley Press.
37. Hsieh, D. A. (1989). Modeling Heteroskedasticity in Daily Foreign Exchange Rates, *Journal of Business and Economic Statistics,* 7: 307-317.
38. Kupiec, P. H. (1995). Techniques for Verifying the Accuracy of Risk Measurement Models, *Journal of Derivatives, winter,* 73-84.
39. Lambert, P., and Laurent, S. (2001). Modelling Financial Time Series Using GARCH-type Models with a Skewed Student Distribution for the Iinnovations, *Univ. Li`ege, Belgium, Working paper.*
40. Laurent, S. and Peters, J.-P. (2002). G@rch 2.2: An Ox Package for Estimating and Forecasting Various ARCH Models, *Journal of Economic Surveys,* 16(3): 447-485.
41. Nelson, D. B. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach, *Econometrica*, 59(2): 347-370.

42. Nyblom, J. (1989). Testing for the Constancy of Parameters Over Time, *Journal of the American Statistical Association,* 84: 223-230.

43. Palm, F. (1996). *GARCH Models of Volatility*, in Handbook of Statistics, ed. By G.Maddala, and C.Rao, Amsterdam: Elsevier Science. 209-240.

44. Palm, F., and Vlaar, P. JG. (1997). Simple Diagnostics Procedures for Modeling Financial Time Series, *Allgemeines Statistisches Archiv,* 81: 85-101.

45. Pagan, A. (1996). The Econometrics of Financial Markets, *Journal of Empirical Finance,* 3: 15-102.

46. Peters, J.-P. (2001). Estimating and Forecasting Volatility of Stock Indices Using Asymmetric GARCH Models and (Skewed) Student-t Densities, *Mimeo, Ecole d'Admin. des Affaires, Unv.of Li`ege*

47. Puttonen, V. (1995). International Transmission of Volatility between Stock and Stock İndex Future Markets, *Journal of International Financial Markets, Institutions & Money,* 5.(2/3).

48. Saltoğlu, B. (2003). *A High Frequency Analysis of Financial Risk and Crisis: An Empirical Study on Turkish Financial Market,* Istanbul: Yaylım Publishing.

49. Sarma, M., Thomas, S. and Shah, A. (2001). Selection of Value-at-Risk Models, *Mimeo*

50. Tang, T.-L., and Shieh, S.-J. (2006). Long-Memory in Stock Index Futures Markets: A Value-at-Risk Approach, *Phsica A*, 366: 437-448.

51. Tsay, R. S. (2005). *Analysis of Financial Time Series,* New York: John Wiley & Sons.

52. Tse, Y. (1998). The Conditional Heteroscedasticity of the Yen-Dollar Exchange Rate, *Journal of Applied Econometrics,* 193: 49-55.

53. Taylor, S. (1986). *Modeling Financial Time Series*, New York : John Wiley & Sons.

54. Wu, G. (2001). The Determinants of Asymmetric Volatility, *The Review of Financial Studies,* 14(3): 837-859.

55. Zakoian, J.-M. (1994). Threshold heteroskedascity Models, *Journal of Economic Dynamics and Control*, 15: 931-955.